NATIONAL INSTITUTE OF TECHNICAL TEACHERS TRAINING AND RESEARCH
(DEEMED TO BE UNIVERSITY UNDER DISTINCT CATEGORY)
CHANDIGARH

Ph.D. (Mathematics) Entrance Examination – December 2025 Session

Subject / Branch / Department :	**Mathematics - Applied Science**
Roll No. :	
Candidate Name :	
Date of Examination :	

Maximum Marks: 25 (There is no negative marking)

Notes: (a) Only one option is to be tick-marked out of the four options given as an answer
(b) The Candidate must put his/her signature with date at the bottom of each page
(c) For any rough work, please use ONLY the back-sides of pages which are left blank

Q1.	How many elements does the set $\{Z \in C \mid z^{60} = -1, z^k \neq -1, for\ 0 < k < 60\}$
(a)	24
(b)	30
(c)	32
(d)	45

Q 2.	The Charpit's equation for the PDE $up^2 + q^2 + x + y = 0, p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial y}$ are given by
(a)	$\frac{dx}{-1-p^3} = \frac{dy}{-1-qp^2} = \frac{du}{2p^2u+2q^2} = \frac{dp}{2pu} = \frac{dq}{2q}$
(b)	$\frac{dx}{2pu} = \frac{dy}{2q} = \frac{du}{2p^2u+2q^2} = \frac{dp}{-1-p^3} = \frac{dq}{-1-qp^2}$
(c)	$\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$
(d)	$\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$

Q3.	Let $p(z) = a_0 + a_1 z + ... + a_n z^n$ and $q(z) = b_1 z + b_2 z^2 + ... + b_n z^n$ be complex polynomials. If a_0, b_1 are non-zero complex numbers, then the residue of $\frac{p(z)}{q(z)}$ at 0 is equal to-
(a)	$\frac{a_0}{b_1}$

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(b)	$\frac{b_1}{a_0}$
(c)	$\frac{a_1}{b_1}$
(d)	$\frac{a_0}{a_1}$
Q4.	Let S be the set of 3×3 real matrices A with $A^T.A=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$. Then the set S contains -
(a)	a nilpotent matrix
(b)	a matrix of rank one
(c)	a matrix of rank two
(d)	a non-zero skew symmetric matrix
Q5.	Let G be a nonabelian group. Then, its order can be -
(a)	25
(b)	55
(c)	125
(d)	35
Q6.	The number of subfields of a field of Cardinality 2^{100} is -
(a)	2
(b)	4
(c)	9
(d)	100
Q7.	An urn has 3 red and 6 balck balls. Balls are drawn at random one by one without replacement. The probability that the second red ball appears at the fifth draw is -
(a)	$\frac{1}{9!}$
(b)	$\frac{4!}{9!}$
(c)	$\frac{4(6!\,4!)}{9!}$
(d)	$\frac{6!\,4!}{9!}$
Q8.	Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta,1)$, where $\theta \in \{1,2\}$. Then which of the following statements about the maximum likelihood estimator (MLE) of θ is correct –
(a)	MLE of θ does not exist
(b)	MLE of θ is $\overline{X}$
(c)	MLE of θ exists but it is not $\overline{X}$

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(d)	MLE of θ is an unbiased estimator of θ

Q9. A group G is generated by the element x, y with the relations $x^3 = y^2 = (xy)^2 = 1$, the order of G is -

(a)	4
(b)	6
(c)	8
(d)	12

Q10. Consider the M/M/1 queue with the arrival rate λ and service rate μ with $\mu > \lambda$. What is the probability that no customer exited the system before time 5 ?

(a)	$\frac{\mu e^{-5\lambda} - \lambda e^{-5\mu}}{\mu - \lambda}$
(b)	$e^{-5\lambda} - e^{-5\mu}$
(c)	$e^{-5\lambda} + (1 - e^{-5\lambda})\frac{e^{-5\mu}}{5\mu}$
(d)	$e^{-5\mu} + (1 - e^{-5\mu})\frac{e^{-5\lambda}}{5\lambda}$

Q.11 Let $W_1 = \{(u,v,w,x) \in R^4, u+v+w=0, 2v+x=0, 2u+2w-x=0\}$ and $W_2 = \{(u,v,w,x) \in R^4, u+w+x=0, u+w-2x=0, v-x=0\}$. Then which among the following is true ?

(a)	$\dim(W_1) = 1$
(b)	$\dim(W_2) = 2$
(c)	$\dim(W_1 \cap W_2) = 1$
(d)	$\dim(W_1 + W_2) = 3$

Q12. Consider the following LPP:

$$Max. Z = x_1 + \frac{5}{2}x_2,$$
$$\text{Subject to } 5x_1 + 3x_2 \le 15,$$
$$-x_1 + x_2 \le 1,$$
$$2x_1 + 5x_2 \le 10, \; x_1, x_2 \ge 0$$

Then the problem –

(a)	Has no feasible solution
(b)	Has indefinitely many optimal solutions
(c)	Has an unique optimal solution
(d)	Has an unbounded solution

Q13. The objective function of the dual problem for the following primal LPP:

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$Max. f = 2x_1 + x_2,$

Subject to $x_1 - 2x_2 \geq 2,$

$x_1 + 2x_2 = 8,$

$x_1 - x_2 \leq 11,$

With $x_1 \geq 0$ and x_2 unrestricted in sign, is given by -

(a)	$Min. z = 2y_1 - 8y_2 + 11y_3$
(b)	$Min. z = 2y_1 + 8y_2 + 11y_3$
(c)	$Min. z = 2y_1 - 8y_2 - 11y_3$
(d)	$Min. z = 2y_1 + 8y_2 - 11y_3$

Q14. The reminder obtained when 16^{2016} is divided by 9 equals -

(a)	1
(b)	2
(c)	3
(d)	7

Q15. The solution of the partial differential equation $u_t - xu_x + 1 - u = 0, x \in R, t > 0,$ subject to $u(x,0) = g(x)$ is

(a)	$u(x,t) = 1 - e^{-t}(1 - g(xe^t))$
(b)	$u(x,t) = 1 + e^{-t}(1 - g(xe^t))$
(c)	$u(x,t) = 1 - e^{-t}(1 - g(xe^{-t}))$
(d)	$u(x,t) = e^{-t}(1 - g(xe^t))$

Q16. If a complete integral of the partial differential equation

$$x(p^2 + q^2) = zp;\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$

Passes through the curve $x = 0, z^2 = 4y,$ then the envelope of this family passing through $x = 1, y = 1$ has -

(a)	z = -2
(b)	z = 2
(c)	$z = \sqrt{2 + 2\sqrt{2}}$
(d)	$z = -\sqrt{2 + 2\sqrt{2}}$

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Q17. Let (x_1, x_2, x_3, x_4) be an optimal solution to the problem of minimizing $x_1 + x_2 + x_3 + x_4$ subject to the constraints $x_1 + x_2 \geq 300$; $x_2 + x_3 \geq 500$; $x_3 + x_4 \geq 400$; $x_4 + x_1 \geq 200$; and $x_1, x_2, x_3, x_4 \geq 0$. Which of the following are not possible values of any x_i ?

(a)	300
(b)	400
(c)	500
(d)	600

Q18. Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first arrival in the queue is a man, is –

(a)	$\frac{\lambda_1}{\lambda_1 + \lambda_2}$
(b)	$\frac{\lambda_2}{\lambda_1 + \lambda_2}$
(c)	$\frac{\lambda_1}{\lambda_2}$
(d)	$\frac{\lambda_2}{\lambda_1}$

Q19. Customers arrive at a sales counter managed by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. What is the average waiting time of a customer –

(a)	225
(b)	200
(c)	250
(d)	150

Q20. Let X and Y be *i. i. d.* random variables uniformly distributed on (0, 4). Then $P(X > Y | X < 2Y)$ is -

(a)	1/3
(b)	5/6
(c)	¼
(d)	2/3

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Q21. The value of integral $\int_{\|z\|=2} \frac{e^{2z}}{(z+1)^4} dz$	
(a)	$2\pi i e^{-1}$
(b)	$\frac{8}{3}\pi i e^{-2}$
(c)	$\frac{2}{3}\pi i e^{-2}$
(d)	0
Q22. The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the Bonus scheme will be introduced if X, Y, and Z become managers are 3/10, ½, and 4/5, respectively. Then, what is the probability that the Bonus scheme will be introduced?	
(a)	23/45
(b)	24/90
(c)	12/45
(d)	23/90
Q23. The joint *p.d.f* of a two-dimensional random variable *(X, Y)* is given by: $f(x,y)=\begin{cases}2, 0<x<1, 0<y<x;\\ 0, \text{elsewhere}\end{cases}$ The marginal density function of X and Y is –	
(a)	2(1 – y)
(b)	2(1+y)
(c)	1-y
(d)	1+2y
Q24. Consider the integral equation $y(x)=x^3+\int_0^x \sin(x-t)y(t)dt,\ x\in[0,\pi]$. Then the value of y(1) is-	
(a)	19/20
(b)	1
(c)	17/20
(d)	21/20
Q25. The radius of convergence of the series $\sum_{n=1}^{\infty} z^{n^2}$ is –	
(a)	0
(b)	∞
(c)	1
(d)	2

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Ph. D Entrance Examination (August 2025)

Ph. D. Mathematics

Correct Answers

1. (c) 2. (b) 3. (a) 4. (a) 5. (b) and (c) both correct

6. (c) 7. (c) 8. (c) 9. (b) 10. (a)

11. (c) 12. (b) 13. (d) 14. (a) 15. (a)

16. (c) and (d) both correct 17. (c) and (d) both correct

18. (a) 19. (a) 20. (a) 21. (b) 22. (a)

23. (a) 24. (d) 25.(c)

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